## MtxVec v6

Users Guide to MtxVec for .NET Framework and .NET Core

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## 1 Introduction

MtxVec numerical library has two main purposes:
1.) Provide framework, which allows cost-efficient development of algorithms that run fast.
2.) Deliver a wide range of ready-to-use algorithms, which execute at impressive speeds out-of-the-box.

This user's guide deals primarily with the intricates of the programing framework. For the list of available ready to use algorithms, please look at the library reference available here:

## https://www.dewresearch.com/Help/VSNET/MtxVec/contents.html

and search the content by Index.

## 2 The framework

By combining all the framework features of the library, it is possible to write fast executing code with minimum effort. To some extend it can be assumed, that if an algorithm can be speeded up, it can be done with this library and that the result will not be much less than the optimal code path in the absolute sense.

For the numerical code to run fast on modern CPUs, the concepts built in to the library address the following:
1.) Computers main memory is a shared resource. When it comes to multi-threading, any shared resource becomes a bottleneck as long as it is shared.
2.) Each CPU core has a dedicated memory called Level 1 cache. If the code is working only with Level 1 cache, there is no resource sharing and the speed can scale linearly with core count.
3.) Only Level 1 cache is fast enough to feed the CPU, when the code is using AVX2 and AVX512 instructions sets. These instructions allow for the fastest possible code.

### 2.1 Reduce memory sharing issues

Memory sharing as a problem grows proportionally with the frequency of memory allocation requests. By reducing the frequency of memory allocation calls, we are also reducing this bottleneck and allow the code to scale better with core count. To reduce this frequency, MtxVec implements for its key objects (TVec/TMtx/TVecInt/TMtxInt) thread-locking free features:
a.) Capacity property. Each time the object is sized below the size of the Capacity, a memory allocation call is saved.
b.) Object cache. A small set of key objects is pre-allocated as a cache in a global structure. Each thread gets its own thread-local cache. Each object comes with pre-allocated memory. It suffices to fetch the object to also get (a preset) amount of ready-to-use memory.
c.) Subranges. It is possible to segment or subrange the memory of key objects. This can be done in-place, not-place and the Subrange calls can be also nested. Matrix memory can be reinterpreted as a vector and vice versa.
d.) For the most part, the library does not create objects or allocate any memory on the heap behind the scenes. If some working memory is needed, this is fetched from the object cache.

### 2.2 Simplifying the use of CPU cache L1

CPU Cache L1 is used, when:
a.) The same continuous block of memory is accessed more than once.
b.) The total size of all blocks accessed does not exceed the size of CPU cache L1 for $90 \%$ of the time. Typical CPU L1 cache size is 32 KB .

This is dealt with in two ways:
a.) Because the key objects (TVec/TMtx/TVecInt/TMtxInt) obtain their memory from the object cache, this results in reusing of the memory blocks at the same physical memory address. Typically, about 5-6 objects can be used within a tight loop not to exceed the size of the CPU's L1 cache.
b.) Additionally, MtxVec implements BlockInit, BlockNext and BlockEnd methods for its key objects. This pattern simplifies sizing of the memory within tight loops while keeping the memory usage below CPU L1 cache size.

### 2.3 Take advantage of the latest CPU instruction sets

The library implements a comprehensive set of "vectorized" math functions like Sin, Cos, Exp, ... Their speed of execution is typically about $10 x$ higher with AVX512 and $5 x$ higher with AVX2 than .NET Core 5.0 built-in Math functions. Nearly all methods of key objects (TVec/TMtx/TVecInt/TMtxInt), which work on array of elements, are vectorized (hardware accelerated). MtxVec will run on all Intel x86 compatible CPU's old and new, but will achieve highest performance on latest CPU generation.

### 2.4 Reducing the cost of code development

MtxVec was designed with simplicity and ease of use in mind:
a.) Object oriented numerical library.
b.) Improved compactness and readability of code when using vectors versus scalars.
c.) Shallow depth of methods/function calls. Simple functions with clear purpose.
d.) Protects the developer from a wide range of possible errors.
e.) XML code comments with code examples embedded in to the source code.
f.) Natural math expression syntax for vectors and matrices with full support for operator overloading for vectors, matrices and complex numbers.
g.) Parameters are explicitly range checked, before they are passed to the dll routines. This ensures that all dll calls are safe to use.
h.) All MtxVec functions must pass strict automated tests. It is these tests, which give the library the highest possible level of reliability, accuracy and error protection.
i.) Many routines are multi-threaded, including 1D FFT, sparse matrix solvers, matrix multiply, and large parts of Lapack.

### 2.5 LAPACK and BLAS library

MtxVec makes extensive use of Lapack. Lapack is short for Linear Algebra Package and was originally called Linpack. Lapack is today de-facto standard for linear algebra and is free (www.netlib.org). Because Lapack is standard, different CPU makers provide performance optimized versions of Lapack to achieve maximum performance. Because linear algebra routines are the bottleneck of many frequently used algorithms, Lapack is a part of code that makes most sense to optimize. MtxVec uses the Lapack in two flavors:
1.) On Windows and Linux a hardware accelerated version provided by Intel with their Math Kernel library.
2.) A version written entirely in $\mathrm{C} \#$ for maximum portability available on all platforms.

Many LAPACK functions take many parameters. Most of them can be filled-in automatically by MtxVec, thus reducing the time to study each function extensively, before it can be used. When calling

Lapack routines, MtxVec automatically compensates for the fact that in FORTRAN the matrices are stored by columns and in other languages by rows.

## 3 Organization

MtxVec library is organized in to three levels - computational level, objects level and component level. The interface to the computational level is a set of class static functions, which are declared in Dew.Math.Units.nmkl, Dew.Math.Units.ippspl, and other classes and implemented as external DLLs MtxVec.Lapack6d.dII, MtxVec.Dsp6d.dII, MtxVec.Sparse6d.dII, MtxVec.FFT.dII, MtxVec.Random.dII and MtxVec.Vmld.dIl. The first level is not documented. The second level introduces vector and matrix objects, complex numbers and a number of class static utility functions declared in:

Dew.Math.Units.MtxVec, Dew.Math.Units.Polynoms, Dew.Math.Units.Math387,
Dew.Math.Units.Probabilities, Dew.Math.Units.SpecialFuncs, Dew.Math.Units.Optimization, Dew.Math.Units.Toeplitz, Dew.Math.Units.Sparse, Dew.Math.TeeChart.

The second level is the "run-time" part of the MtxVec library. Third level is formed by a set of components which are built on the second level to offer a centralized and quick access to large parts of the library many times also offering ready to use user interface.

This user's guide concentrates mostly on the second level, specifically the classes declared within the main Dew.Math namespace, Dew.Math.Units.Math387 class, Dew.Math.Units.MtxExpr class and touches some features from Dew.Math.TeeChart class. It gives a good overview on the concept and core features of MtxVec .

### 3.1 Compiler support

MtxVec v6 for VS.NET supports the following compilers and frameworks:

| Visual Studio C\#, VB.NET, C++ | 2015. NET. 2017.NET, 2019.NET |
| :--- | :--- |
| .NET Framework | 2.0 to 4.8 |
| .NET Core | 5.0 and newer |

MtxVec is available in two variants:

- Dew.Math.dll. These variant references .NET Framework or .NET Core 5.0 with Winforms support. Depends on external higher performance dlls and runs on Windows only. The managed code is written in C\#.
- Dew.Math.Core.dIl. References .NET Core 5.0 and is cross-platform capable. This variant is written $100 \%$ in C\#. On Linux OS only, this variant references external unmanaged high performance libraires, which are shipped with the product.

MtxVec version 6.0.4 is the last version with support for .NET framework. Future versions will be based only on .NET Core.

## 4 Quick start

```
using Dew.Math;
using Dew.Math.Units;
using Dew.Math.Editors;
using Dew.Math.Controls;
using Dew.Math.Tee;
private void button1_Click(object sender, EventArgs e)
{
    TCplx ac = "1+2i";
    Matrix am = MtxExpr.RandGauss(5, 5, true); //5x5 complex matrix with Gaussian
    Vector av = MtxExpr.Ramp(25); //real vector = [0, 1, 2,...., 23, 24]
    Matrix bm = am * av + ac + 2; // (*,/) treat Matrices as vectors
                            // Same as: ./ and .* operators (not linear algebra)
    MtxVecEdit.ViewValues(bm,"Complex matrix",true); //display the matrix contents
// To make linear algebra multiplication and divison use functions
    bm = MtxExpr.Divide(am,bm) + 2; //matrix divison and add 2 to all elements
    bm = MtxExpr.Mul(am,bm) + 3; //matrix multiply
//of course you can mix matrices and vectors
    double[] arr = null;
    av.Resize(5,false);
    av = MtxExpr.Mul(av,bm) + 2; //vector from left and matrix multiply
    av.CopyToArray(ref arr); //copy "complex" data to array of double length
    Vector bv = (Vector) arr; //copy data from array to vector,
    MtxVecEdit.ViewValues(bv, "Vector from array", true); //display the vector
    for (int i = 0; i < am.Rows; i++) //standard loop example
    {
        av.Values[i] = Math387.Real(am.CValues[i,0]*av.Values[i]);
    }
}
```


## 5 MtxVec programming interface

### 5.1 Object hierarchy

MtxVec organizes mathematical data structures and methods in to objects to simplify memory management and increase ease of use and features the following class hierarchy:


Figure 1 MtxVec class hierarchy
TVec, TVecInt and TMtx classes are encapsulated with Vector, Vectorlnt and Matrix classes:


Figure 2 Mapping to value classes (struct).
All methods and properties of TVec, TVecInt and TMtx are also accessible from Vector, Vectorint and Matrix types which are declared as value classes (struct). Vector and Matrix are not simply derived from TVec and TMtx because in C++ Vector and Matrix are value classes, for which it is not necessary to call constructors explicitly. Because that is not also a feature of C\# language, for all practical purposes, Vector and Matrix behave as if though they are derived from TVec and TMtx.

### 5.2 Mathematical expressions and operator overloading

Custom operator overloading means that the programmer can use $/,{ }^{*},-,+$ operators also for his own types and not only with built in types. To support operator overloading MtxVec declares types: Vector and Matrix. The usage of the new types is best demonstrated with an example:

```
private void Test()
{
    TVec c = new TVec();
    TVec b = new TVec();
    b.Size(1000);
    b.RandUniform(0.5, 1.5);
```

Probabilities.BoseEinsteinPDF (b, 0.3, 0.1, c); \}

When using Vector objects:

```
private void Test()
{
    Vector c = new Vector(0);
    b = MtxExpr.RandUniform(1000, 0.5, 1.5);
    Probabilities.BoseEinsteinPDF(b, 0.3, 0.1, c);
}
```

All properties and methods of TVec and TMtx are mapped to Vector and Matrix value classes including Values and CValues array properties. The performance of the code using Vector and Matrix objects is on pair with TVec and TMtx for longer vectors (1000 elements and beyond). The +,-,/ and * operators are strictly per element operations (+. , -. , /., *.). To perform matrix multiplication or division use the Mul and Divide functions.

You can also mix TVec/TMtx and Vector/Matrix in the same expression. An expression will accept TVec and TMtx as a variable only, if the variable next to it is of Vector or Matrix type:

```
private Vector Test(TVec p)
{
    Vector bv = new Vector(0);
    bv.Copy(p);
    Vector Result = bv * p; //same as Result.Sqr(p);
    return Result;
}
```

Dew.Math.Units.MtxExpr declares also functions which return Vector or Matrix as a result. These are equivalent to the methods of TMtxVec and its descendants. For example, the following calls give the same result:

```
private void Test()
{
    Vector bv = new Vector(0);
    Vector av = new Vector(0);
    TVec avt = new TVec();
    avt.Sin(av);
    bv.Sin(av);
    Vector cv = MtxExpr.Sin(av);
}
```

All those methods of TVec and TMtx are also available as functions in MtxExpr.pas, where the result of the function is a single Vector (TVec) or a single Matrix (TMtx).

### 5.2.1 Element by element Vector/Matrix expression types

Multiply two vectors element by element:

```
Vector bv = new Vector(0);
Vector av = new Vector(0);
/ /..
Vector cv = av*bv;
//Both vectors av and cv must have the same length or an exception will be raised;
//either or both of the vectors can be complex.
```

To multiply vector and matrix element by element:

```
Matrix bm = new Matrix(0,0);
Vector av = new Vector(0);
```

```
//..
Vector cv = av*bm;
//Both av vector and the bm matrix must have the same number of elements or an
exception //will be raised;
//either or both of the av and bm can be complex.
```

The same principles are used for addition (+), substraction (-) and divison. It is of course possible to mix Matrix and Vector types with integers, doubles, reals and complex numbers in the expressions.

### 5.2.2 Linear algebra Vector/Matrix expression types

Matrix, Matrix multiply: $C=A$ * $B, \quad C=\operatorname{MtxExpr} \cdot \operatorname{Mul}(A, B)$;

The TMtx.Mul method also features parameters which make it possible to implicitly transpose or adjungate one or both matrices.

Matrix, Vector multiply: $b=A^{*} x$,
Vector, Matrix multiply: $b=x^{*} A$,
Vector, Vector multiply: $A=b^{*} x$,

| Vector, Matrix divide | : $\mathrm{b}=\mathrm{x} / \mathrm{A}$, | $\mathrm{b}=$ MtxExpr.Divide $(x, A)$; |
| :---: | :---: | :---: |
| Matrix, Matrix divide | : $\mathrm{B}=\mathrm{C} / \mathrm{A}$, | $B=$ MtxExpr.Divide(C,A); |
| Vector, Matrix Idivide | : b = Alx, | $\mathrm{b}=$ MtxExpr.LDivide $(\mathrm{A}, \mathrm{x})$; |
| Matrix, Matrix Idivide | : $\mathrm{B}=\mathrm{A} \backslash \mathrm{C}$, | $B=$ MtxExpr.LDivide(A,C); |

### 5.2.3 Linear algebra with TVec and TMtx types

When working with TVec vectors and TMtx matrices it is important to remember, that in most cases the type of the result defines the object which has that method. A few examples:

Matrix, Matrix multiply: $\mathrm{C}=\mathrm{A} * \mathrm{~B}, \quad$ C.Mul(A,B);

The TMtx.Mul method also features parameters which make it possible to implicitly transpose or adjungate one or both matrices.

Matrix, Vector multiply: $b=A$ * $x$, b.TensorProd $(A, x)$;
Vector, Matrix multiply: $b=x$ * $A$, $\quad b$.TensorProd $(x, A)$;
Vector, Vector multiply: $A=b^{*} x$, A.TensorProd( $b, x$ );
Sparse matrix, vector multiply: $B=S$ * $b, \quad$ S.MulRight( $b, B$ );

The sparse matrix has that method and it returns the result in the second parameter. Sparse matrix is in this case an exception to the initial rule.

Vector, sparse matrix multiply: $B=x^{*} S, \quad$ S.MulLeft( $x, B$ );
Matrix, sparse matrix multiply: $B=A^{*} S$, $\quad$ S.MulLeft(A, $B$ );
Sparse matrix, matrix multiply: $B=S^{*} A, \quad$ S.MulRight(A,B);

## Other types of operations

Matrix addition or subtraction is straightforward with the Add method. Sparse matrix features specialized routines for this purpose also. To obtain a diagonal of a matrix there is a Vector.Diag method. To set a diagonal of a matrix there is a Matrix.Diag method. To get a row/column of the matrix call Vector.GetRow (Vector.GetCol) and to set one: Matrix.SetRow. (Matrix.GetCol).

Many other methods follow the same pattern. The exceptions are usually methods which return multiple variables as a result and their overloads. A few examples: Matrix.Eig, Matrix.SVD, Matrix.LQR.

### 5.2.4 Implicit type conversions

Implicit type conversions can help clean up the code. The string can be auto magically converted to a complex number:

TCplx ac = "1+2i";
When a function requires an array of double, Vector or Matrix can be passed instead. Implicit type conversions will result in dereferencing Vector.Data.Values1D pointer which is an array of double. Explicit type conversions of Vector/Matrix to double[] or TCplx[] will result in a copy operation.

Passing Vector or Matrix to a function accepting TVec or TMtx will pass the contained object.

```
Matrix am = MtxExpr.RandGauss(5, 5, true); //5x5 complex matrix with Gaussian
Vector av = MtxExpr.Ramp(25); //real vector = [0, 1, 2,...., 23, 24]
Matrix bm = am * av + ac + 2; // (*,/) treat Matrices as vectors
    // Same as: ./ and .* operators (not linear algebra)
// To make linear algebra multiplication and divison use functions
bm = MtxExpr.Divide(am,bm) + 2; //matrix divison and add 2 to all elements
bm = MtxExpr.Mul(am,bm) + 3; //matrix multiply
//of course you can mix matrices and vectors
double[] arr = null;
av.Resize(5,false);
av = MtxExpr.Mul(av,bm) + 2; //vector from left and matrix multiply
av.CopyToArray(ref arr); //copy "complex" data to array of double length
Vector bv = (Vector) arr; //copy data from array to vector,
```


### 5.3 Method conventions

TMtxVec classes and descendants have a list of methods which may be called like this:

```
vector_object_a.Add(vector_object_b);
```

The object can hold either real or complex double precision data. If the method can put its result in one object, then the result is placed in the object on the left. In the following example the result is placed in the objects on the right:

```
a.CplxToReal(Real, Imag);
a.CartToPolar(Amplt, Phase);
```

It's useful to remember this when mixing TVec and TMtx types. A matrix operation which has TVec type as result will be a part of the TVec class and a vector operation which has a TMtx type result will be a part of TMtx class.

### 5.4 Range checking

All methods and properties of TMtxVec descendants are explicitly "range checked". Range checking ensures that the user can not read or write values past the size of the allocated memory. Once the code is compiled without assertions, range checking is disabled and higher performance can be achieved in some cases. Every effort has been made to prevent the user of the library to make an error that would result in memory overwrite. (Writing or reading to parts of the memory which were not allocated before and thus overwriting data of another part of the application.)

Important: Only the release version of MtxVec is compiled without assertions and will perform at maximum speed. After the installation, the release version is located in a separate "release" folder.

### 5.5 Making use of the abstract class

Many methods can accept any TMtxVec descendant class:
TVec. Copy (TMtxVec Src);
When a parameter is of TMtxVec or TDenseMtxVec type, the function will accept TVec, TMtx, Vector and Matrix types. This is one of the most powerful features of MtxVec:

- When the source is vector, the vector size and its data are simply transferred to the calling object.
- When the source is 2 D matrix, the Rows and Cols information is lost and the entire matrix is copied as if it is a vector.
- When the source is a sparse matrix, only the non-zero elements are copied, while the nonzero sparse pattern and the number of rows and columns is lost.

The following versions will also work flawlessly:
TMtx. Copy (TMtxVec Src);
TSparseMtx.Copy(TMtxVec Src);
but with one slight difference. If the source is of the same type as the destination, the method also sets the size of the destination object:

- When the source and destination are TMtx (2D matrix), the method sets Rows, Cols and Complex property of the destination and copies all data values from the source.
- When the source and destination are TSparseMtx (2D sparse matrix), the method sets Rows, Cols, non-zero sparse pattern and Complex property of the destination and copies all data values from the source.
- When the source and destination are TVec (1D vector), the method sets the Length and Complex properties.

If the source and destination are not of the same type, the data is copied as if the source is a vector. No exception is raised only, if the source and the destination have a matching Length and Complex properties.

If only the complex property is to be changed, but all the other properties describing the data preserved, the following method can be called:

```
TMtxVec.Size(TMtxVec Src, bool aComplex);
```

Note: The size method does not preserve the data in the destination object. This is not needed because the destination is overwritten anyway. To resize the object and keep existing data call the Resize method.

To allow such level of abstraction, the TMtxVec class introduces several methods that allow working with the data of descendants as if it was a simple one dimensional array of values:

```
TMtxVec.Values1D - array property to access real values
aTMtxVecObject.Values1D[1] = 1; //sets real value at index I to I
//aTMtxVecObject can be TVec, TMtx, Matrix or Vector
```

```
TMtxVec.CValues1D - array property to access complex values
aTMtxVecObj.CValues1D[1] = Math387.Cplx(1, 0); //sets complex value at index 1 to 1
```

TMtxVec.PValues1D - function returns a pointer to real value

```
IntPtr aPointer;
aPointer = aTMtxVec.PValues1D(1); //returns a pointer to value at index 1
```

TMtxVec.PCValues1D - function returns a pointer to complex value

```
IntPtr aPointer;
```

aPointer $=$ aTMtxVec.PCValues1D(1); //returns a pointer to complex value

### 5.5.1 Writing abstract class code

This is best examined by an example. The following method can accept TVec, TMtxVec, Vector or Matrix. Dst however does not have to have the preset size:

```
private void CustomExpj(TMtxVec Dst, SrcOmega)
{
    Dst.Size(SrcOmega,true);
    CustomExpjNoSize(Dst, SrcOmega);
}
```

The "abstract magic" is achieved by calling the Size method. This method is "virtual" and implements all the required behavior when setting the size of the destination. CustomExpjNoSize in the last line just fills the destination with the result. It is important to note the Size method allows the user to change the Complex property without knowing the actual object type and by preserving all other property values. The True flag passed to the Size method sets the Complex property to True and is optional (default is false).

Of course not all functions can accept abstract object types. For those that don't it is possible to narrow down the required type to either TVec, TMtx or TSparseMtx. If the function should accept only TVec and TMtx, but not TSparseMtx, request that the parameter should be of TDenseMtxVec type. Methods and properties that are to be used for abstract MtxVec code:

- Pointers (IntPtr): PValues1D, PCValues1D, PIValues1D,
- Getting/settings values: Values1D, CValues1D, IValues1D
- Setting size: Complex, Length, Size(Src: TMtxVec, IsComplex: boolean);
- all methods of TMtxVec class.

By making use of the TVec.SetSubRange method, virtually any TVec method can be applied to the source data:

```
private void CustomExpj(TMtxVec Dst, SrcOmega)
{
    Vector a = Vector(0);
    Dst.Size(SrcOmega,true);
    a.SetSubRange(Dst);
    a.Expj(SrcOmega); //call a TVec only method here
}
```


### 5.6 Function parameters

It is recommended to declare parameters of methods as TMtxVec, TDenseMtxVec, TVec, or TMtx and not of Vector or Matrix type. Regardless if the Vector or Matrix type is passed with a ref parameter or not, they will always behave as if passing an object; by reference. It is also possible to mix TVec/TMtx and Vector/Matrix types in expressions and to copy an array of doubles to Vector:

```
private void Test1(TVec v)
private void Test2()
{
    Double[] av;
    test1((Vector) av)); //this will work, but not by reference
    //..
}
```

The array will be not be passed by reference. Any changes made to v, will not be visible in av. That is because the explicit typecast to Vector created a new Vector variable that copied the data from the array in to the new temporary variable.

### 5.7 Indexes, ranges and sub-ranges

Most TMtxVec methods support indexing. Here is a typical pattern that can be observed throughout the library:

```
public TMtxVec Exp();
public TMtxVec Exp(TMtxVec X);
public TMtxVec Exp(int Index,int Len);
public TMtxVec Exp(TMtxVec X, int XIndex,int Index,int Len);
```

The first function version takes no parameters. The result overwrites the source data. The source data can be either real or complex and the method will apply the appropriate code to compute the result. The second version first checks the size of the source objects and tries to match the destination object to be of the same size. If the size operation is successful, the appropriate code is applied to compute the result.( See chapter 5.5 on how the size operation is performed).

The third version takes only Len values starting at Index. If the object is a 2D matrix and has 10 rows and 13 columns, its Length property is 130 . The routine check's if Index and Len are within limits and applies the Exp function only to Len elements starting at position Index. To apply the Exp function to all elements within the matrix, the Index would be set to 0 and Len would be set to 130. Except for some exceptions, most indexed methods (methods that have Index and Len as a parameter) will raise an exception if the destination does not have a matching value of the Complex property.

One other important thing to mention about fourth version of the function is that the destination size is never changed. The only function version changing the size of the destination is the second. Both third and fourth function versions just perform error checking. An easy to remember rule: All methods taking Index and Len parameters never change the size of the destination. For vector this means Length, for the matrix rows and cols properties and for the sparse matrix rows, cols and nonZeros. There are some exceptions that allow changing the value of the complex property. Add, Sub, Mul, Div, Offset and Scale methods allow mixing of real and complex data even for indexed methods. (This was new in v2.0). If the result is to be complex, but the destination stores values of real type, all the destination values that are not to be overwritten will be converted to complex numbers with imaginary part set to zero. These automatic conversions are done in the most optimal way possible.

More examples:
a.Copy (b, 2,0,10);
'a.Copy' means 'copy' 10 elements of "b" from index 2 to "a" starting at index 0 of a. If there are no index parameters, the size of the target object will be set automatically. An alternative means for indexing is to use SetSubIndex or SetSubRange methods:

```
b.SetSubRange (2,10);
a.Copy(b) ;
```

Which is the same as:

```
b.SetSubIndex (2,11);
a.copy(b);
```

The use of SetSubRange and SetSubIndex is recommended because it employs memory reuse, which takes advantage of the CPU cache, which in turn improves performance. SetSubRange can be called on object itself or it can obtain a view of memory from another object:

```
b_vec.SetSubRange(aMatrix,2,10);
```

a.Copy (b_vec);

This is the same as:

```
a.Size(10);
a.Copy(aMatrix,0,2,10);
```

There are other types of indexing where there is a need to apply an operation to specific noncontinuous indexes within a vector or matrix. This can result in heavy performance penalties (heavy means by a factor of 100) for some numerical algorithms. The entire CPU architecture is based on the assumption that memory is accessed by consecutive memory locations in about $90 \%$ of cases. It is therefore best to first gather the scattered data into one dense vector, perform math operations and then scatter the gathered data back to the original location:
a.Gather (b, null, TIndexType.indIncrement, 2, 0) ;
a. Log10();
a.Exp();
a.Scatter (b, null, TIndexType.indIncrement, 2,0 ) ;

This code will copy every second element from b to a , apply math and then scatter the result back to b without affecting other values in b. The Gather and Scatter methods can also accept an index or a mask vector. To access elements of an index vector via IValues array:

```
a.IValues[0] = 1;
```

IValues points to the same memory as Values and CValues and uses a simple integer array type pointer. Although TMtxVec can hold an array of integers, there are no functions that support operation on integers other than copy operations. There are more routines that can help with scattered data:
a.FindMask(b, "=", c);

The method will return ones for all indexes where $b$ and $c$ have a matching value and zeros elsewhere. FindAndGather can be used to find all indexes within $b$ where values are different from NAN (not a number) and apply processing only to those values:
a.FindAndGather (b, "<>", Math387.NAN, Indexes);
a.Scale(2);
b.Offset (1);
a.Scatter (b, Indexes) ;

### 5.8 TVec and TMtx methods as functions

Ideally mathematical expression is written with Vector and Matrix records like this:
$a=a^{*} b+b ;$

For TVec and TMtx this can not be done. The closest syntax allowed is this:

```
a.Add(c.Mul(a,b),b); //where a,b,c are TVec objects
```

Almost every method of TVec and TMtx returns "this" or the Self. This allows nesting of calls like in the example above. Syntax like this can result in a memory leak, if TVec is allocated in unmanaged memory:
$\mathrm{a}=\mathrm{c} \cdot \mathrm{Mul}(\mathrm{a}, \mathrm{b}) ; / / d o n^{\prime} \mathrm{t}$ do this for TVec/TMtx

### 5.9 Complex data

Both Vector (TVec) and Matrix (TMtx) can hold real and complex data. Here is an example:

```
a.Length = 10;
a.Complex = true;
```

a.Length now becomes 5 . Setting the complex property will simply halve or double the length property of the vector. The allocated memory will not change. There is a need however to view that memory as a real or as a complex array:

```
a.Values[0] = 1;
a.CValues[0] = Cplx(2,3);
```

a.Values[0] now becomes 2, because both Values and CValues arrays point to the same memory. The only difference between them is that one is of type double and the other is of type TCplx (TCplx = struct Re,Im: double; end;).

Real and complex Vectors and Matrices and TCplx variables can be mixed together in expressions:

```
public Vector GetSomething()
{
    Vector av = new Vector(10); //vector size
    TCplx ac = "1+2i"; //Convert from string to complex number
    Vector Result = av + ac + 2; //always by value operations
    // ./ and .* (not linear algebra)
    return Result;
}
```


### 5.10 MtxVec types

MtxVec declares many different types, but some should be mentioned explicitly:

### 5.10.1 TMtxFloatPrecision

This type is used everywhere where it is neccessary to declare floating point precision. By default, the value is "mvDouble", but can also be specified to be "mvSingle", "mvDoubleComplex" and "mvSingleComplex".

### 5.10.2 TCplx

Declared as:

```
public struct TCplx
{
    public double Re;
    public double Im;
}
```

This is the default complex number type used by MtxVec. Many object-oriented libraries declare the complex type as an object type (public class Complex)). This means that the consecutive elements within the array are then no longer stored at consecutive memory locations, which affects performance. TCplx has overloaded all the necessary operators and it is possible to write nearly any expression the same as with real valued numbers.

Even though Complex type was introduced in C\# since .NET framework version v4 it has inherited some inefficiencies even in .NET core.

### 5.10.3 Complex number arrays

Internally the complex numbers are stored in arrays of double or arrays of float elements. C\# struggles with code efficiency when dealing with structs. All complex number math is done on arrays, which store real numbers.

## 6 Accessing values of Vector and Matrix

### 6.1 Array property access

Example:

```
Matrix am = new Matrix(5, 5);
TMtx am2 = new TMtx();
am.Values[0, 0] = 2;
am2.Values[0, 0] = 3;
```

Array properties allow a clean range checked access. The array properties perform explicit range checking.

### 6.2 Handling of complex data:

Default array property access is not available for complex data because the object can have only one default array property. Default array properties allow the property name to be left out:

```
TMtx a = new TMtx();
a[1, 0] = 2;
instead of:
```

```
a.Values[1,0] = 2;
```

The following access methods are semantically equivalent:
a.CValues $[1,0]=\operatorname{Cplx}(2,0) ; \quad\{$ Cplx is a function that returns a TCplx struct type

## 7 Memory management

### 7.1 Introduction

The memory for the Matrix is allocated by setting the Rows and Cols properties:

```
Matrix am = new Matrix(0,0);
a.Rows = 4; //allocates nothing
a.Cols = 4; //a now holds 16 elements
a.Rows = 0; // deallocates memory
a.Size(4,4,False); //same as: a.Complex = false; a.Rows = 4; a.Cols = 4;
```

The complex property should be set before setting the Cols property. All arrays are zero based. (The first elements is always at index 0 ).

There are some special issues that need to be taken in to account when working matrices. TMtx/Matrix interfaces highly optimized FORTRAN code. There are two more properties available from TMtx and Matrix:

```
a.Values1D[i]
a.CValues1D[i]
```

Pointers behind these two properties point to the same memory location as Values and CValues pointers. But instead of accessing the elements by rows and columns, they see the whole matrix as a one-dimensional array. To access matrix elements:

```
a1 = a.Values1D[i*Cols+j];
```

This will access the same matrix element as:

```
a1 = a.Values[i,j];
or
a1 = a[i,j];
```

The preferred method for memory allocation is by using the Size method:

```
a.Size(4,4,false,false);
```

Size method will ensure that no more memory is allocated than necessary when resizing. Imagine a $5 \times 10000$ matrix, being resized to $10000 \times 5$, but the rows are set to 10000 first creating a matrix with $10000 \times 10000$ elements, possibly causing an out of memory message.

Matrix data is stored in row-major ordering of C and PASCAL and not in column major ordering of the FORTRAN language. All appropriate mappings are handled internally.

### 7.2 Using TVec and TMtx from cache

TVec and TMtx are always created explicitly. They are never created inside a function and returned as result by MtxVec library or any of its add-on packages. Although some functions return TVec and TMtx these are only methods of TVec and TMtx which return "this" or "self". Furthermore TVec and TMtx can be created in two different ways:

1. From the managed memory where the memory is garbage collected:
```
private void Test()
{
    TVec c = new TVec();
    TVec b = new TVec();
    b.Size(1000);
    b.RandUniform(0.5, 1.5);
    Probabilities.BoseEinsteinPDF(b, 0.3, 0.1, c);
}
```

2. From cache memory, where the memory has to be freed explicitly, but giving zero pressure on the garbage collector logic.
```
private void Test()
{
    TVec a;
    TVec b;
    MtxVec.CreateIt(out a, out b);
    try
    {
        b.Size(1000);
        b.RandUniform(0.5, 1.5);
        Probabilities.BoseEinsteinPDF(b, 0.3, 0.1, c);
    }
    finally
    {
        MtxVec.FreeIt(ref a, ref b);
    }
}
```

Object cache is a set of objects, which are created when the application is started. When a call to Createlt is made, no object actually gets created. The Createlt procedure simply assigns a pointer to an already created object to the parameter. That is not all since the already created object has some
(unmanaged) memory allocated and there is no new memory allocated until some default size is exceeded. In compare to the garbage collector, it can sometimes deliver speed gains of 5 x or more because it increases memory reuse, and thus effectiveness of the CPU cache. The memory allocated by Createlt is never moved as long as the allocated size is smaller than specified cache size. You can freely mix operations between TVec/TMtx and Vector/Matrix.

Matrix and Vector types can be created implicitly and internally to procedures and functions which provides a more programmer friendly user interface, but at the same time, the programmer loses the control over the memory allocation and its performance related cost.

```
private void Test()
{
    Vector c = new Vector(0); //note that Vector(0) constructor is called (!)
    b = MtxExpr.RandUniform(1000, 0.5, 1.5);
    Probabilities.BoseEinsteinPDF(b, 0.3, 0.1, c);
}
```

This fine level of memory allocation control can be very beneficial when there is a need to optimize the code for speed (See Benchmarks chapter).

### 7.3 In-place/not-in-place operations

In case of very large matrices the memory requirements would become a problem ( $10000 \times 10000$ matrix requires 800 MB storage). In such cases the user can use LAPACK routines directly by adding Dew.Math.Units.Nmkl to the uses clause. Whenever possible LAPACK performs matrix operations inplace. Often the matrix size can be greatly reduced by using banded matrix format or sparse matrices.

## 8 Range checking

The array range checking is not performed by the compiler for matrices, but TMtx does perform explicit range checking:

```
a[i, j] = 2; //The property checks the indexes i and j to be within the bounds
```

This additional range checking is enabled when the code is compiled with assertions turned on. When the assertions are disabled, the additional range checking is also disabled and higher performance can be obtained.

## 9 Why and how NAN and INF

NAN is short for Not a Number and INF is short for infinity. CLR does not raise an exception when a division by zero occurs or an invalid floating operation is performed. This allows code in loops without the checks, if the function input parameters are within the definition area of that function. This alone speeds up the code, because most of the try-except and if-then clauses used for that purpose can be left out. Instead, you can concentrate on the code itself and let the CPU work out the details. If a division by zero occurs and floating point exceptions are off, then the FPU (floating point unit) will return INF (for infinity). If divide zero by zero is attempted, the FPU will return a NaN (not a number). When working with arrays, this can be very helpful, because the code will not break when the algorithm encounters an invalid parameter combination. It is not until the results are displayed in the table or drawn on the chart that the user will notice that there were some invalid floating point combinations. It might also happen that INF values will be passed to a formula like this: number/INF (= 0 ) and the final result will be a valid number.

MtxVec offers specialized class static routines for string to number and number to string conversions in Math387 class (StrToVal ,StrToCplx, FormatCplx, FormatSample, StrToSample, SampleToStr) and drawing routines in MtxVecTee class (DrawValues, Drawlt) capable of handling NaN and INF values. By using those routines, the user will avoid most of the problems when working with NAN and INF values. StrToSample for example will convert a NAN or INF string to its floating point presentation:

```
double a = Math387.StrToSample("NAN");
if (Math387.IsNaN(a)) throw new Exception("a = NAN");
```

StrToFloat routine would raise an exception on its own. To test for a NAN and INF value, the first attempt would look like this:

```
if (a = Math387.NAN) ...
```

This however will not work. NAN and INF are not values which are defined with all the bits of a floating point variable. There are just a few bits that need to be set within a floating point variable which will make it a NAN or an INF. The proper way to test for a NAN and INF are therefore these:

```
if (IsNaN(a)) ...
if (IsInf(a)) ...
if (IsNanInf(a)) ...
```

MtxVec methods and routines correctly handle NAN and INF. It is therefore acceptable to write something like this:

```
if (a.Find(Math387.Nan) > 0) ..//test if "a" vector holds a NAN value
```


## 10 Serializing and streaming

### 10.1 Streaming with TMtxComponent

All components should be derived from a common ancestor: TMtxComponent. (Declared in MtxBaseComp.pas). This component features the following methods:

- SaveToStream
- LoadFromStream
- SaveToFile
- LoadFromFile
- Assign
- AssignTemplate
- LoadTemplateFromStream
- SaveTemplateToStream
- LoadTemplateFromFile
- SaveTemplateToFile

What is interesting about TMtxComponent is that all components derived from it have all their published properties streamed, without the need to make any changes to the five routines. Therefore, all components derived form TMtxComponent have the capability to store their states (properties) to the stream, file or to assign from another object of the same type.

The "template" routines set a protected property named BlockAssign to true. Property setter routines can then prevent properties to be changed. This is very useful when there is a need to save only "parameters" and not the "data" of the component. The parameters will remain the same, while the "data" will be different next time and there is no point in wasting disk space by saving it.

Important: With release of MtxVec v4 for VS.NET these methods of TMtxComponent are not yet functional.

### 10.2 Streaming of TVec, TMtx and TSparseMtx

All TVec/TMtx and Vector/Matrix types are marked as [Serializable]. TVec and TMtx also have their own methods for streaming:

## SaveToStream

LoadFromStream

SaveToFile
LoadFromFile
Assign
These routines will process all the published properties and data of the TVec and TMtx objects and will save and load data only in the binary format. (MtxVec also supports Matrix Market text file format.) However, sometimes it is necessary to read a text file. Here is how this can be done:

### 10.3 Write TMtx/Matrix to a text file

```
Matrix amtx = new Matrix(20, 20,true);
amtx.RandUniform(-1,2);
TStringList StringList = new TStringList();
amtx.ValuesToStrings(StringList, "\t","",""); // use tab = (char) 9 as delimiter
StringList.SaveToFile("C:\\ASCIIMtx.txt"); // Save matrix values to txt file
```


### 10.4 Read TMtx/Matrix from a text file

```
Matrix amtx = new Matrix(0,0);
TStringList StringList = new TStringList();
StringList.LoadFromFile("C:\\ASCIIMtx.txt"); // Load matrix values from txt file
amtx.StringsToValues(StringList, "\t"); // use tab = (char) 9 as delimiter
MtxVecEdit.ViewValues(amtx) ;
```


## 11 Input-output interface

### 11.1 Reading and writing raw data

TVec and TMtx have the capability to save their state via SaveToStream and LoadFromStream. The downside on using these two routines is that it is not possible to save the raw data only. The values of all the properties are always included as the header of the saved data block. Saving raw data only can be achieved by using two other methods: TVec.WriteValues and TVec.ReadValues. These two methods will read and write to and from System.IO.Stream descendants only the contents of Values array itself (raw data). When writing to stream, it is also possible to define the precision and consequently the size of the disk space to occupy. Supported precisions include:
enum TPrecision \{
prDouble, prSingle, prInteger, prCardinal,
prSmallInt, prWord, prShortInt,
prByte, prMuLaw, prALaw, prInt24
\}
MuLaw and ALaw are audio compression standards for compressing 16 bit data to 8 bits. prlnt24 is a 24 bit signed integer useful for 24bit digital audio.

When data is being read with ReadValues, the type of the data must be explicitly specified. All data types are converted to double.

### 11.2 Formatting floating point values - FloatToString

Math387 class declares the following routines:

```
public string FormatCplx(TCplx Z, string ReFormat, string ImFormat);
public string FormatSample(double X, string Format);
```

Both are similar to FormatFloat and return a string representing the floating point number passed as the first parameter. If the Format parameter is an empty string, the routines will call the SampleToStr and CplxToStr routines. They are declared like this:

```
public string SampleToStr(double X, int Digits, int Precision);
```

and are similiar to FloatToStr. The Digits parameter specifices the minimum number of digits in the exponent. The Precision defines the number of digits to represent the number. Example:

```
double a;
string myString;
a = 12.123456;
myString = Math387.SampleToStr(a, 0, 3); // myString = '12.1';
a = 12.123456;
myString = Math387.SampleToStr(a, 0, 15); // myString = '12.123456';
```

To convert from string to a floating point number, the following routines can be used:

```
public TCplx StrToCplx(string Source);
public double StrToSample(string Source);
```

They are similar to system.double.Parse with a few exceptions when it comes to handling NAN and INF values. (See the chapter: "Why and how NAN and INF".)

### 11.3 Displaying the contents of the TVec and TMtx

### 11.3.1 Displaying data as a delimited text

Vector and Matrix have two methods for getting and setting their values to and from a text file. They are called StringsToValues and ValuesToStrings. You can use Dew.Math.TStringList class together with Vector or Matrix ValuesToStrings method to write Vector or Matrix values to Dew.Math.TStringList string array:

```
using Dew.Math;
using Dew.Math.Controls;
TStringList strarray = new TStringList();
Matrix aMtx = new Matrix(3, 5);
aMtx.RandGauss();
aMtx.ValuesToStrings(strarray,"\t", "
    0.#####;-0.#####", "+0.#####i;-0.#####i");
```

You can also write values to single text line:

```
aMtx.Size(5, 3, true);
aMtx.RandGauss(3, 1);
string txt = aMtx.ValuesToText("\t", "
    0.#####;-0.#####", "+0.#####i;-0.#####i");
```

In both cases the methods support:

- handling of complex and real values,
- accept NAN and INF values,
- can get/set only a sub vector or a sub matrix,
- control the displayed precision.


### 11.3.2 Displaying data within a grid.

The best way to display the contents of a matrix is within a DataGridView control. For this you can use Dew.Math.GridServices ValuesToGrid, GridToValues and PrepareGrid routines OR Dew.Math.Controls.MtxGridView control.

For small and medium object size (few fundred matrix columns, rows or vector elements) the preffered way is to use MtxGridView control:

```
using Dew.Math;
using Dew.Math.Controls;
// create complex matrix
Matrix m= new Matrix(20, 5, true);
m.RandGauss();
// Create MtxGridView and connect it to Matrix m
MtxGridView mtxGridView1 = new MtxGridView();
mtxGridView1.DataObject = m;
```

MtxGridView control allows you easy viewing/editing of real or complex matrices or vectors. The control is derived from DataGridView, inherits all it's properties/methods and introduces full support for complex numbers. This means that any changes to MtxGridView cell values are automatically propagated to Matrix and Values.

Another way to connect values to DataGridView is to use Dew.Math.GridServices routines. This approach offers even more customization, but it lacks the automation MtxGridView has built-in. The following code copies all values from matrix to DataGridView:

```
using Dew.Math;
using Dew.Math.Controls;
// create complex matrix
Matrix m= new Matrix(20, 5, true);
m.RandGauss();
// Grid should be 20 rows x 10 columns, two colums per complex number
GridServices.PrepareGrid(dataGrid1,20,10);
GridServices.ValuesToGrid(m,dataGrid1,0,0);
```

In this case changes in dataGrid1 are not automatically updated to Matrix m Values. This has to be done by using GridServices.GridToValues routine:

```
using Dew.Math;
using Dew.Math.Controls;
Matrix m;
// Grid is 20 rows x 10 columns, two colums per complex number,
// no headers
GridServices.GridToValues(dataGrid1,m, true, false);
```

Check Dew Math demo and help files for differences between two approaches.

### 11.3.3 Viewing data by using built-in Values editor

Dew.Math.Editors.MtxVecEdit class provides several routines which allow easy viewing/editing of matrix/vector values. By calling the Dew.Math.Editors.MtxVecEdit.ViewValues routine a simple editor will be displayed allowing the user to examine the values of a Vector or Matrix object.

```
using Dew.Math;
using Dew.Math.Editors;
Vector v = new Vector(100);
v.RandGass();
MtxVecEdit.ViewValues(v)
```



This editor can be displayed modally or not. If it is displayed modally, the values can be changed and the contents of the object will also change. If the editor is not displayed modally, the changes will be discarded. If the changes are not to be saved, then the user can freely select the number formatting. If the changes are to be saved, the number formatting must be full precision or otherwise the values will be truncated to the displayed precision. The values can not be edited unless Editable flag from the View menu is checked.

Editor status bar shows basic object info shown, in this case real vector of size $100 \times 1$ elements. Header columns and header rows show object cell indices, starting with 0 . Additional information in status bar shows which cell is currently being selected and/or edited (in this case first vector element is being selected). Row and column indices can be shown or hidden by using Row Index and Column Index menu items in the View menu.

If object values are complex numbers, then header columns labels are formatted according to Split Complex Numbers menu item value from the View menu. If selected, column header labels are named as "0-Re", "0-Im", "1-Re", "1-Im", ..., where $0-R e$ shows the real part of the first cell complex number, " $0-\mathrm{Im}$ " shows the imaginary part of the first complex number, etc... If not selected then each column shows properly formatted object cell complex number.

### 11.4 Charting and drawing

Vector or matrix values can also be drawn directly on the chart by calling the TeeChart.Drawlt routine. This class is located in the Dew.Math.Tee namespace. TeeChart class contains a large set of class static DrawValues routines. This routines copy data from TVec or TMtx to the defined TChartSeries. Adding of new values is optimized for the TeeChart version used and charting can be considerably faster, if DrawValues is used. DrawValues routines also take care of any NAN's and INF's.

```
Vector a;
a.LoadFromFile("c:\test.vec");
MtxVecEdit.ViewValues(a); //display a window showing values in "a"
TeeChart.DrawIt(a); //display a chart of values in "a"
```

On Figure 3 the magnitudes of the values stored in the complex matrix can be seen. The layout of the values is the same as in the matrix editor (the left axis labels should be inverted).


Figure 3

## 12 Programming style

Every programmer has a preferred style of programming: different indentation, different variable naming and different coding style (use of exceptions, for loops, dynamic memory allocation etc.). This section lists some recommendations.

### 12.1 Mixing TVec/TMtx and Matrix/Vector types.

Handle Vector/Matrix as objects even though they are of struct type. When writing new functions and methods, continue to declare their parameters as TVec or TMtx, because Vector and Matrix will be implicitly converted (dereferenced) to TVec and TMtx anyway.

### 12.2 Try-finally blocks and unmanaged memory

Every time a call is made to Createlt/Freelt or Create/Destroy pair to allocated cached memory, it should be placed within a try-finally block like this:

```
TVec a1;
TVec b1;
MtxVec.CreateIt(out a1,out b1); //work vectors
try
{
}
finally
{
    MtxVec.FreeIt(ref a1, ref b1);
}
```

These have two purposes:
If there is an exception within the try-finally block, the allocated objects and memory will be freed and the program user will be able to retry the calculation with other parameters. It is now easier to track what is created and what is destroyed, because it is clearly visible where create and where destroy is called. MtxVec has internal variables tracking the state of the object cache.

Do not write code like this:

```
MtxVec.CreateIt(out b);
//...some code here
MtxVec.CreateIt(out a);
yourProc(a,b);
MtxVec.FreeIt(ref b);
//.. some code here..
MtxVec.FreeIt(ref a);
```

This makes it difficult to see, if all calls to Createlt have Freelt pairs. It is also a good rule of houskeeping to group the code allocating the memory separately from the code doing calculations. This makes the code much more readable.

### 12.3 Raising exceptions

Because all code is now protected with try-finally blocks, exceptions can be raised safely to indicate an invalid condition. When the user tries to perform calculation with an MtxVec application, this is what will happen:

1. Allocate memory for the calculation.
2. Start calculation
3. Display results.
4. Free allocated memory and resources.

If during the calculation an error condition is encountered, because the data is not valid, raising an exception will first Free allocated memory and resources and then display a message box stating what the error was. It is important that memory was freed, because now the user can retry the calculation with new data or parameters, without the need to restart the application to reclaim the lost memory.

### 12.3.1 Invalid parameter passed:

$\{$

```
    if (a.Length != b.Length) throw new Exception("a.Length <> b.Length");
    //...
```

\}

This will pass an exception to the higher-level procedures, which will free any allocated memory and exit. Once the exception reaches the highest-level routine a message box will be displayed with text "a = null" and an OK button.

### 12.4 When to use Createlt/Freelt

All objects created within a routine should be destroyed within that same routine. If TVec or TMtx are global objects, make them a part of an object or component. Global objects are those, which are not created and destroy very often and might persist in memory throughout the life of an application. This rule should be followed in order not to waste the object cache. The purpose of object cache is to allow speedy memory allocation and deallocation. Where this is not needed, it should not be used, because that could slow down other routines using it:

- Object cache might run out of precreated objects and calls to Createlt/Freelt would result in direct calls to Create/Free.
- Object cache size would have to be increased to prevent (1) and the entire application would require more memory.
- Within garbage collected environment it is best if Createlt/Freelt pairs used within the same function as stack variables and not as fields of objects. This simplifies the usage, where the programmer does not need to care about freeing the objects outside of the same function scope.


## 13 Getting up to speed - the benchmarks

### 13.1 Floating point code vectorization

MtxVec also allows the programmer to write high level object code that gives the benefits of the most optimized assembler version of the code supporting latest CPU instructions from within your current development environment. This is best examined on an example. Simply trying to use a faster Power function in the following loop will bring no major gains:

```
for (i = 0; i < 1000000; i++) do
{
    Y[i] = (c1*Ax[i]+c2)/Math. Power(1.0 + Math.Power(Bx[i],eA), eB);
}
```

But if the above loop is rewritten like below things change a lot.

```
a.Length := 2000;
b.Length := 2000;
for (i = 0; i < 500; i++) do
{
    YourFunc(a,b,c1,c2,ea,eb);
}
//By using expressions:
public Vector YourFunc(Vector a,Vector b,double c1,double c2,double ea,double eb)
{
    Vector Result = (c1*A+c2)/MtxExpr.Power(1.0 + MtxExpr.Power(B,ea), eB);
    return Result;
}
//Using TVec:
public void YourFunc(TVec a,TVec b,TVec Result,double c1,double c2,double ea,
double eb)
{
    TVec a1;
    TVec b1;
    if (a.Length != b.Length) throw new Exception("a.Length <> b.Length");
    MtxVec.CreateIt(out a1,out bl); //work vectors
    try
    {
        a1.Copy(a);
        a1.Scale(c1);
        a1.Offset(c2);
        b1.Power(b,ea);
        b1.Offset(1);
        Result.Power(b1,-eb);
        Result.Mul(a1);
    }
    finally
    {
        MtxVec.FreeIt(ref a1, ref b1);
    }
}
```

We can note that we wrote more lines and that we create and destroy objects within a loop. The objects created and destroyed within the function are not really created and not really destroyed. The Createlt and Freelt functions access a pool of precreated objects called object cache. The objects from the object cache have some memory pre-allocated. But how could so many loops, instead of only one, be faster? We have 7 loops (Copy, Scale, Offset, Power, Offset, Power, Mul) in the second case and only one in the first. This makes it impossible for any compiler to perform loop optimization, store local variables in the CPU/FPU, precompute constants. The secret is called SIMD or Single Instruction Multiple Data. Intel's and AMD CPU's support a special instruction set. It has been very difficult for any
compiler vendor to try to make efficient use of those instructions and even today most compilers run without support for SIMD with two major exceptions: Intel C++ and Intel Fortran compilers. SIMD supporting compilers convert the first loop of our case in to the second loop of our case. The transformation is not always as clean and the gains are not as nearly as large, as if the same principle is employed by hand. Sometimes it is difficult for the compiler to effectively brake down one single loop in to a list of more effective ones.

What is so special about SIMD and why are more loops required? The SIMD instructions work similar to this:

- load up to 4 array elements from memory (ideally takes 1 CPU cycle)
- execute the mathematical operation (ideally takes 1 CPU cycle)
- save the result back to memory(ideally takes 1 CPU cycle)

Total CPU cycle count is 3 . The normal loop would require 1 cycle for each element to load, store and apply function (in best case). In total that would be 12 CPU cycles. Of course the compiler does some optimization in the loop, stores some variables in to FPU registers and the loop does not need full 12 cycles. Therefore typical speed ups for SIMD are not $4 x$ but about $2-3 x$. However there are some implicit optimizations in our second loop too. Because we know that the exponent is fixed, the vectorized Power function can take advantage of that, so the gap is increased again. Of course, the first loop could also be optimized for that, but you would have to think of it.

### 13.2 Block based processing

When working with vectors it is absolutely critical to also consider the size of the CPU cache. If the arrays will not fit in the available CPU cache, a large (sometimes up to $3 x$ ) performance penalty will be imposed upon the algorithm. This means that vector arithmetic's should not be applied to vectors whose size exceed certain maximum length. Typically the maximum number of double precision elements ranges from 800 to 2000 per array. Longer vectors have to be split in pieces and processed in parts. MtxVec provides tools that allow you to achieve that easily. The following listing shows three versions of the same function.

Plain function:

```
public double MaxwellPDF(double x, double a)
{
    double xx;
    xx = x* x;
    return Math.Sqrt(4*a*Math387.INVTWOPI)*a*xx*Math.Exp(-0.5*a*xx);
}
```

Vectorized with expressions:

```
public Vector MaxwellPDF(Vector x, Vector a)
{
    Vector xx = x * x;
    Vector res = Math.Sqrt(4 * Math387.INVTWOPI * a) *
                        a * xx * MtxExpr.Exp (-0.5 * a * xx);
    return res;
}
```

Vectorized function:

```
private void MaxwellPDF(TVec X, double a, TVec Res)
{
    TVec res1;
    MtxVec.CreateIt(out res1);
    try
    {
```

```
        res1.Sqr(X);
        res.Mul(res1, -0.5 * a);
        res.Exp();
        res.Mul(res1);
        res.Mul(Math.Sqrt(4 * Math387.INVTWOPI * a) * a);
    }
    finally
    {
        MtxVec.FreeIt(ref res1);
    }
}
```

Block vectorized function:

```
private void MaxwellPDF(TVec X, double a, TVec Res)
{
    TVec res1;
    MtxVec.CreateIt(out res1);
    try
    {
        Res.Size(X);
        X.BlockInit();
        res.BlockInit();
        while (!X.BlockEnd)
        {
            tmp.Sqr(X);
            res.Copy(tmp);
            res.Mul(-0.5 * a);
            res.Exp();
            res.Mul(tmp);
            res.Mul(Math.Sqrt(4 * a * Math387.INVTWOPI) * a);
            res.BlockNext();
            X.BlockNext();
            }
    }
    finally
    {
        MtxVec.FreeIt(ref res1);
    }
}
```

On Core i7 860 (2.8 GHz, Quad Core 2 CPU ) the vectorized function is about $5 x$ faster than the plain function (Figure 4). The block vectorized version of the function is a little slower for short vectors but maintains its high performance even for vectors exceeding 10000 double precision elements.


Figure 4 Benchmark
The block vectorized function is only marginally faster than vectorized version due to the use of SSE2/SSE3/SSE4/AVX instructions. If the CPU does not support SSE, then the gain of the block vectorized version will be much more significant (typical gains are about 6 times). For example, when using older CPU's the speed of the plain function for vectors with length larger than the size of the CPU cache will be higher than that of its vectorized version. The vectorized version has to access memory multiple times, while the plain function version can cache some intermediate results in to FPU registers or CPU cache. The block vectorized version will ensure that the chunk of the vector being processed can fit in to the CPU cache and will thus give optimal performance for long vectors even in that case. The green line shows the behavior of the Vector/Math expressions. Below 320 elements the plain function is much faster, however still $4 x$ slower than vectorized version, which uses Createlt and Freelt to allocate memory and does not use vectorized math expressions (only TVec and TMtx).

### 13.3 Garbage Collector and High Performance numerical math

To measure the effect of the garbage collector on the high performance numerical algorithms we designed a special test and compared impact on both the classical not vectorized functions and on vectorized functions. In both cases, the computational part of the timed loop and the part where the memory was allocated were separate and distinct code parts:

1. No memory was allocated inside the timed loop that was used by the computational part. In fact, the computational part was not allocating nor releasing any managed memory inside the timed loop.
2. All memory allocated in the computation part was non movable (GC pinned or unmanaged)
3. The GC memory allocation part of the algorithm was not doing any computation.

Such a setup can show how would performance of a numerical algorithm, be affected by memory allocations in the same or some other part of the program.

The amount of memory allocated is a parameter to the benchmark. Figure 5 is showing the results and the code for the vectorized version is listed below. The $X$ axis shows the length of the input vector $X$ and the Y axis is time normalized per element. The absolute cost of the garbage collector in our case for arrays longer than 1024 elements is approximately $20-30 \mathrm{~ms}$, or $100 \%$. (The complete source code is part of the MtxVec demo under "Memory allocation->Garbage collector benchmark".)


Figure 5 Garbage collector cost for GCIterCount = 5 (see source code)

```
private void MaxwellPdf1(TVec X, double a, TVec Res)
{
    TVec res1;
    MtxVec.CreateIt(out res1);
    try
    {
        res1.Sqr(X);
```

```
        res.Mul(res1, -0.5 * a);
        res.Exp();
        res.Mul(res1);
        res.Mul(Math.Sqrt(4 * Math387.INVTWOPI * a) * a);
    }
    finally
    {
        MtxVec.FreeIt(ref res1);
    }
}
private double MaxwellNoBlock(int Iterations) {
    double a = 1;
    double[] testArray;
        int counter = Environment.TickCount;
        for (int i=0;i<Iterations;i++) {
            MaxwellPdf1(x, a, res);
            for (int k = 0; k < GCIterCount; k++) //GC Loop
            {
                testArray = new double[res.Length];
                testArray[1] = resArray[1];
            }
        }
        return Environment.TickCount - counter;
}
```

An analysis about the type slowdown gives an answer, if the slowdown occurred due to the garbage collector execution cost, or was the numerical part of the code somehow affected by the garbage collector like invalidation of the CPU cache.

Lessons learned: Memory should be allocated with care inside tight high performance loops because the processing time rises considerably with the amount of memory allocated.

The code that needs really high performance, should pre-allocate memory before entering the loop; when inside the loop it should allocate minimum amount of arrays and use Createlt/Freelt where possible. From Figure 4 we have also learned, that for vectors shorter than about 600 elements, we should be careful not to allocate them repeatedly inside tight loops for best performance:

```
for (Int32 j = 0; j < 10000000; j++)
{
    TVec a = new TVec(); //don't do this
    / /..
    a.Size(20); //this is OK. Calling a.Size does not reallocate if same size
}
```


### 13.4 Common pitfalls

1. When using block vectorization, make sure that the temporaries are "not" block vectorized. Only the input vector and the output vector are block vectorized. In the example with MaxwellPDF, it would be unnecessary to call Blocklnit on Res1 TVec object. Typically block vectorization would be the last optimization to perform for the application and it would be applied to the top level function only. This allows the programmer to control the size of the blocks that are being processed throughout the algorithm from one central point. This is why no functions from TVec, TMtx, TDenseMtxVec and TMtxVec have been block vectorized.
2. Vectorization also increases the use of memory. Keeping the vectors short, will keep the memory usage low.
3. The default size of the block for vectors storing complex numbers should be less than 512 , or it should not exceed the size of vector memory preallocated by object cache.
4. About $98 \%$ percent of functions available are SSE2/SSE3/SSE4.2/AVX vectorized but not all. It makes sense to have an algorithm available even if it is not executing with the highest performance. Specifically, the following functions are not vectorized: all variants of Find (including Findlndexes, FindMask etc...), and some complex number versions of certain functions. The user is best advised to check the source code if in doubt. Future versions will include more vectorized functions. One way to get better performance with these functions is to make sure that block processing rules are always observed.
5. The trigonometric functions are extensively used in complex number math. It is important to be aware of some limitations of real valued sine and cosine functions. Their performance depends upon the size of the argument: $\sin (1)$ will be computed faster than $\sin (100000)$. This is true for standard FPU instructions and for the SSE versions. Together with the speed, the accuracy of the sine/cosine will be reduced also. If the number has 20 digits, only the last 10 numbers after the decimal point will remain valid. Math387 class includes a utility function called FixAngle which should be called on the argument before it is passed to the sine function, but only if a "large" argument is to be expected. This will fix the accuracy and the speed problem. Of course, if the argument is not large, the speed will decrease and the accuracy will not be improved.
6. There are penalties on processing NAN and INF. Make sure those are fished out from the vectors soon after they might occur, to lower the performance cost on the follow up code.
7. For CPU's without SSE, the only way to improve the performance is to strictly follow the rules of block processing. (CPU cache size).
8. When running "quick" benchmark tests make sure to pass "valid" parameters to functions. If the function is not defined in a region, the result of the function will be a NAN or INF. All subsequent functions that will receive NAN or INF at the input could run much slower. This is the limitation of the SSE instructions. The program must be guarded against such cases explicitly. (Either by checking the user input or by inserting checks in the code that will abort the algorithm sooner if a NAN or INF is detected.). This can be important when running algorithms that already take a long time to compute with valid data.
9. Intel also warns about denormals. They are another cause for slowdown. Denormals are numbers which get truncated due to limited floating point number range. So again, the input to the algorithm when testing it, should be valid data.

## Vectorized expressions specifics:

10. Your compiler may not support a specific optimization called: collection on common sub expression. If your expression contains the same expression multiple times, be sure to assign its result to a temporary variable to prevent the expression from being evaluated multiple times.
11. Use parenthesis to indicate which part of the expression should be evaluated first. There is no optimization analysis that would be based on precedence of the same operator. Namely, multiplying two vectors or multiplying two real numbers is very different in terms of clock cycles used. But the compiler does not know that.

Garbage collector specifics:
12. The processing cost of the garbage collector can be high. MtxVec will deliver best performance when used in accordance with design guidelines for GC.

### 13.5 Code vectorization methods

Vectorizing the code means writing the code with the help of vector and matrix variables. Only when the code is written in such form has the compiler or the underlying library a chance to exploit the SIMD instruction set. With the evolution of CPU's such design will be bringing increasingly bigger gains over traditional code design. Vectorizing the code will provide by far the greatest performance boost keeping multi-core gains far behind in the shade. Therefore, if there is any chance to vectorize the code, it should by all means be attempted.

When vectorizing the code we have to rewrite all our functions so that they take input data in vector form and work around if-then sentences. If-then sentences are in-fact the biggest party breaker when it comes to code vectorization and it also makes sense to show an example of a method called "Vector patching", which allows very effective vectorization of a great deal of additional code.

Many times the code vectorization is limited by a few special values which have to be handled separately. If the if-then's handling them cannot be moved out of the loop, make another loop following the first in which you only check for the special values using standard if-then sentences. Because the first computationally intensive loop has been vectorized, the extra loop patching up the vector is a really cheap way out. Below is an example of the Power function with vector patching. Notice that the vectorized part is followed by a separate loop patching up the result.

```
function TMtxVec.Power(Base: TMtxVec; Exponent: TCplx): TMtxVec;
    { X^Y, X >= 0 }
var a,b: TVec;
        i: integer;
begin
    Result := Self;
    if Self = Base then raise EMtxVecInvalidArgument.Create ('Self = Base');
    Size(Base, TRUE);
    if Math387.Equal(Exponent,0) then
    begin
        Result.SetVal(C_ONE);
        Exit;
    end;
    if fLength = 0 then Exit;
    if Base.Complex then
    begin
            vzPowx(Base.PCValues1D(0),Exponent,PCValues1D(0),Base.Length);
            for i := 0 to fLength-1 do
            begin //Resolve the special cases
                if Math387.Equal(Base.CValues[i],0) then
                    begin
                        if Exponent.Im = 0 then
                        begin
                        if Exponent.Re > 0 then CValues[i] := C_ZERO else
                            if Exponent.Re < O then CValues[i] := C\overline{p}lx(Inf) else
                                    CValues[i] := C_ONE;
                        end else CValues[i] := CNAN;
            end;
        end;
    end else
    begin
            CreateIt(a,b);
            try
                a.Ln(Base);
                b.Mul(a,Exponent);
                Exp (b);
                for i := 0 to fLength-1 do
                begin //Resolve the special cases
                if Base.Values[i] < 0 then
                begin
                    CValues[i] := Math387.Power(Cplx(Base.Values[i]),Exponent);
                end else
                if Base.Values[i] = 0 then
```

```
                                    begin
                        if Exponent.Im = 0 then
                        begin
                if Exponent.Re > 0 then CValues[i] := C_ZERO else
                    if Exponent.Re < O then CValues[i] := C\overline{plx(Inf) else}
                                    CValues[i] := C_ONE;
                            end else CValues[i] := CNAN;
                    end;
            end;
        finally
            FreeIt(a,b);
        end;
        end;
    GCOperation(Base);
end;
```

This will work only if the special values are "rare". If you have to distribute the processing down multiple paths nearly equally, things get complicated. In this case, one way out is to split the vector in to sub vectors and store the indices of individual values within the vector separately. Once the processing of each separate part has completed, merge the individual parts together again. Good starting points for this approach are TVec.Gather and TVec.Scatter methods.

### 13.6 Managing the threads

The threads launched by MtxVec will default to the maximum core count in the system. By default the library decides on its own if, when and how many threads it will launch to speed up the processing. Some parameters however can be customized.

MtxVec declares a global variable inside of MtxVec unit called controller. This controller object has properties to control the object cache and threading. Specifically it is possible to set and read thread count for all subsystems like FFT, BLAS, VML and IPP. Additionally, it is also possible to enable or disable threading by setting ThreadingMode. Other information provided includes CPU cache size, CPU core count and CPU frequency.

Additional two properties are DenormalsAreZero and ThreadWaitBeforeSleep. Denormals are numbers which are smaller than the floating point range and thus clipped in precision. This property controls the CPU flag and affects all processing. This is similar to floating point exception flag, which is disabled by default. Forcing denormals to zero will substantially improve SSE performance, according to Intel by up to $50 x$, if your algorithms occasionally produce denormal values. The downside of this speed up is a small loss of precision.

ThreadWaitBeforeSleep is set to 0 [ms] by default. It affects all threads of all threading subsystems (FFT, BLAS, VML and IPP). Sometimes it is desirable to further decrease thread context switch time at the expense of CPU usage. By setting this value to greater than zero, the threads will not enter sleep once completing a job, but will actively wait for the next job consuming $100 \%$ of the CPU until timeout occurs. This makes it possible to speed up short jobs, which could otherwise not be threaded. The downside is a higher CPU usage than otherwise spent mostly on thread waiting and use of system resources which cannot be spent by other applications.

When executing multiple threads on CPU's which contain multiple multi-core CPU's, like for example Intel Q6600, it becomes important to group threads working on the same data on the same pair of cores. By default MtxVec sets the thread affinity to true internally, to ensure that. On true quad or more core CPU's this setting is obsolete.

## 14 Debugging MtxVec

### 14.1 Viewing the values of Vector and Matrix in the debugger as an array

Hovering with mouse over TVec, TVecInt, VectorInt, Vector or Matrix variable types while debugging will show the values stored inside of these objects as if though they were declared as arrays.

When the debugger watches (Figure 6) are no longer sufficient, it is possible to invoke the debugger visualizer for TVec/TMtx, Matrix and Vector types (Figure 7, Figure 8) by pressing on the magnifying glass icon.

| + av |  | Dew.Math.Vector |
| :---: | :---: | :---: |
| + avi | " $6 \times 1$ (int32) $(3,3,3,3,3,3)^{*}$ | Dew.Math.VectorInt |
| + am | ${ }^{5} 3 \times 3$ ( $-0.98069098,1.8288219,1.5205879$ Q - | Dew.Math.Matrix |

Figure 6 Inspecting contents of Vector "av", "avi" and Matrix "am"

```
Vector av = new Vector(5); //vecto
Matrix am}=\mathrm{ new Matrix (5, 5);
    # / am|V|{Dew.Math.Matrix}
\ Matrix Visualizer
```

Figure 7 Starting matrix visualizer during debugging.

| Matrix values |  |  |  |  | - 回 |  | E3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Edit View Action | Actions |  |  |  |  |  |
| Resize $A=0 \mid \operatorname{Re}(A) \operatorname{Im}(A) \quad A(T) A^{*} \operatorname{Mag}(A)$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| 0 |  | 1 | 2 | 3 | 4 |  |  |
| 0 | 0.059684867843829738 | -0.14195999171311197 | 0.65630999036661608 | 0.20167604233462297 | 0.49486149210574876 |  |  |
| 1 | -0.41855413232607491 | -0.061588493682402969 | -1.2351269158196159 | -0.0041077685400349581 | 0.040596532650035246 |  |  |
| 2 | 0.25794282584526745 | -0.40079528029735256 | 0.96389919336591412 | 1.8251460941332778 | -0.71878306047260909 |  |  |
| 3 | -1.5761995987994923 | 0.21741727199683852 | 0.81602285386036111 | -0.051279269334096038 | 0.0020048082033720009 |  |  |
| 4 | 0.31216179781025949 | -2.5100556612343832 | 0.025795532105576679 | -0.52507916897392093 | 0.1349105781390269 |  |  |
| Matrix: $5 \times 5$ (Real) Row 0, Col 0 |  |  |  |  |  |  | $\therefore$ |

Figure 8 Displayed matrix contents while debugging

### 14.2 Printing current values of variables

The most common way to debug an algorithm in the "old days" was to print the values of the variables to the screen or to the file. Today it is more common to use watches and tooltips to examine the values of variables. In some cases this two approaches do not work because of multithreading. Sometimes it is also desirable to have an algorithm return report on its convergence, like it is the case with optimization algorithms. For cases like this there is a global variable called Report declared in MtxVec class. Report is of a TMtxVecReport type and encapsulates StringBuilder class. It has been extended with several new methods to support:

1. Saving the stream to system.IO.Stream or a file.
2. Write the contents of TVec and TMtx objects to the stream. (as text)
3. Specify the text formatting of the floating point values.

## Typically the Report object can be used like this:

```
Vector av = new Vector(5); //vector size
Matrix am = new Matrix(5, 5);
```

```
av.Ramp();
am.RandGauss();
MtxVec.Report.Print(new TMtxVec[2] { av, am }, new string[2] { "av
vector","am matrix" });
MtxVec.Report.NewLine();
MtxVec.Report.SaveToFile("C:\\TestReport.txt",false);
MtxVec.Report.Clear();
```

The printout will look this:

```
av vector (5x1) =
    0.0000
    1.0000
    2.0000
    3.0000
    4.0000
am matrix (5x5) =
\begin{tabular}{rrrrr}
-1.4301 & -1.7811 & 0.7858 & -0.3901 & 1.7490 \\
0.8135 & 0.1826 & -1.2526 & 0.7739 & -0.3879 \\
0.4901 & 0.3925 & -0.9281 & 0.1107 & 0.4650 \\
0.0928 & -0.3188 & 1.2720 & -0.3946 & 1.0211 \\
0.3266 & -0.0072 & -0.6555 & -0.2365 & 1.5490
\end{tabular}
```

The last parameter in the print command defines the variable name. Vectors and matrices can be mixed within the same Print method call. Other useful methods declared next to those already defined in the TMtxVecReport class are:

- report.PrintVec
- report. PrintMtx
- report.PrintSample
- report.PrintCplx
- report.PrintSampleArray
- report.PrintCplxArray


### 14.3 Calling Drawlt and ViewValues

Of course you can also inspect the contents of objects with the functions provided:

```
Vector av = new Vector(5); //vector size
Matrix am = new Matrix(5, 5);
av.Ramp();
am.RandGauss();
MtxVecEdit.ViewValues(av, "Vector values", true);
TeeChart.DrawIt(av, "Vector values", false);
MtxVecEdit.ViewValues(am, "Matrix values", true);
TeeChart.DrawIt(am, "Matrix values");
```


### 14.4 Memory leaks

When using Vector/Matrix or TVec/TMtx classes, all the memory is managed automatically for the programmer giving him a free ride.

When using unmanaged memory with TVec and TMtx we have to always match Createlt/Freelt pairs, as already emphasized. MtxVec class holds two global variables: Controller.MtxCacheUsed and Controller.VecCacheUsed. Their value will show the number of unfreed objects. After the application has finished using MtxVec routines, these two variables should have a value of 0 . This means that all objects for which Createlt was called, were also passed to the Freelt routine.

The memory preallocation is disabled by calling:
MtxVec.Controller.SetMtxCacheSize(0, 0);
MtxVec.Controller.SetVecCacheSize (0, 0);
The first parameter defines the number of TVec/TMtx objects created in advance and the second parameter defines the number of array elements for which to preallocate the memory. By setting memory preallocation to zero AV's will be raised much closer to the actual cause of the problem.

### 14.5 Memory overwrites

When using TVec and TMtx or Vector and Matrix memory overwrites should be a thing of the past. But if you chose to work directly with unmanaged memory overwrite errors may not pop up immediately. The reason for this is that TVec and TMtx objects residing in the object cache have preallocated a specified number of elements. Such pre-allocation speeds memory allocation for small vectors and matrices considerably and also makes reallocations faster. MtxVec explicitly checks all parameters passed to TVec and TMtx routines for range-check errors.

## 15 Getting ready to deploy

Once the app has been debugged and is ready to be deployed, several files required by MtxVec have to be included in the distribution package. These files are located in the windows\SysWOW64 or windows Isystem32 directory. Since May 2021, the version of the library is embedded in to the name of the library. This allows coexistence of multiple versions on the same computer:

- MtxVec.Lapackd.X.Y.Z.dII. This library is mandatory and contains lapack functions.
- MtxVec.Dspd.X.Y.Z.dII. This library is mandatory and contains many vectorized math functions.
- MtxVec.Vmld.X.Y.Z.dIl. This library contains many vectorized math functions from IPP. They are smaller in size than MKL versions. The dll is linked in by default.
- Libiomp5md.dIl. This library is mandatory and contains OPENMP runtime used by other multithreaded dlls.


### 15.1 Compact MtxVec

Compact MtxVec is an initiative to allow the customer to provide additional processing muscles only where needed, and at the same time keep the distribution size as low as possible:

- MtxVec.Random.X.Y.Z.dll is used by the random generators located in RndGenerators class. The random generators are threaded and vectorized.
- MtxVec.Vml.X.Y.Z.dll contains vectorized and threaded math functions like sin, cos, exp, etc...
- MtxVec.FFT.X.Y.Z.dII. This library contains multithreaded 1D, 2D and 3D DFT and FFT functions.

The provided dlls have specialized code of each function for at least Intel Core Duo compatible CPU, and for SSE3/SSE4/AVX/AVX2/AVX512 instruction sets. The appropriate code version is selected automatically, when the libraries are loaded.

## 16 64bit version of MtxVec

MtxVec can run either as 32bit or 64bit. When run in 64bit mode it features noticeably faster floating point math ranging in average from $20-50 \%$ in compare to its 32 bit counterpart. The decision to run as either 32bit or 64bit is made by the applications settings specifying any, x86 or x64 as the target OS. To run MtxVec in 64bit mode, you need to copy 64bit versions of the required dll's to System32 dir on a 64bit OS. 32bit dll's are located inside SysWOW64 dir and have the same name as their 64bit counterparts.

The 64bit mode used uses 32bit integers and 64bit pointers. This means that you can allocate at most 16GB large arrays for double precision and 8GB large arrays for single precision. This is especially useful for LAPACK and sparse matrix algorithms.

## 17 Major function groups

The following function groups do not contain all the functions, but they do allow a faster navigation when searching for most common routines when writing custom functions. The "features" column in the tables can contain the following keywords:

SSE3/SSE4/AVX/AVX512 - instruction sets supported
SMP - symmetric multiprocessing (support for multiple CPU's)
RCX - allows mixing real and complex numbers in the same expression even for indexed versions.
All functions also accept complex data where applicable. The math expression column is usefull when writing a custom function and some expressions can be grouped for faster execution.

### 17.1 Basic vector math

| Function | Class | Math expression | Features |
| :---: | :---: | :---: | :---: |
| Abs | TMtxVec | $\begin{aligned} a[i] & =\|a[i]\| \\ a[i] & =\|b[i]\| \end{aligned}$ | SSE3/SSE4/AVX |
| Add | TMtxVec | $a[i]=a[i]+B$ | SSE3/SSE4/AVX, RCX |
| Add | TDenseMtxVec | $\begin{aligned} & a[i]=b[i]+c[i] \\ & a[i]=a[i]+S^{*} c[i] \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { RCX } \end{aligned}$ |
| AddProduct | TDenseMtxVec | $a[i]=a[i]+b[i] * *[i]$ | SSE3/SSE4/AVX |
| ArcCos | TMtxVec | $\begin{aligned} & a[i]=\operatorname{ArcCos}(a[i]) \\ & a[i]=\operatorname{ArcCos}(b[i]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \\ & \hline \end{aligned}$ |
| ArcCosh | TMtxVec | $\begin{aligned} & a[i]=\operatorname{ArcCosh}(a[i]) \\ & a[i]=\operatorname{ArcCosh}(b[i]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| ArcCot | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{ArcCot}(\mathrm{a}[i]) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{ArcCot}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \\ & \hline \end{aligned}$ |
| ArcCoth | TMtxVec | $\begin{aligned} & a[i]=\operatorname{ArcCoth}(a[i]) \\ & a[i]=\operatorname{ArcCoth}(b[i]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| ArcCsc | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{Arccsc}(a[\mathrm{i}) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{ArcCscs}(\mathrm{b}[\mathrm{ij}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| ArcCsch | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{Arccsch}(\mathrm{a}[\mathrm{ij}) \\ & \mathrm{a}[\mathrm{ij}=\operatorname{ArcCsch}(\mathrm{b}[\mathrm{ij}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| ArcSec | TMtxVec | $\begin{aligned} & a[i]=\operatorname{ArcSec}(a[i]) \\ & a[i]=\operatorname{ArcSec}(b[i]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| ArcSech | TMtxVec | $a[i]=\operatorname{ArcSech}(a[i])$ <br> $a[i]=\operatorname{ArcSech}(b[i])$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| ArcSin | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\operatorname{ArcSin}(a[i]) \\ & \mathrm{a}[i]=\operatorname{ArcSin}(\mathrm{b}[\mathrm{i}) \mathrm{a} \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| ArcSinh | TMtxVec | $a[i]=\operatorname{ArcSinh}(a[i])$ <br> $a[i]=\operatorname{ArcSinh}(b[i])$ | SSE $3 /$ SSE4/AVX, SMP |
| ArcTan | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\operatorname{ArcTan}(a[i]) \\ & a[i]=\operatorname{ArcTan}(\mathrm{b}[i]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| ArcTan2 | TMtxVec | $a[i]=\operatorname{ArcTan2}(\mathrm{b}[\mathrm{i}], \mathrm{c}[\mathrm{i}])$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| ArcTanh | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{ArcTanh}(\mathrm{a}[\mathrm{ij}) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{ArcTanh}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Cbrt | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\left(\mathrm{a}[\mathrm{i} 1)^{1 / 3}\right. \\ & \mathrm{a}[\mathrm{i}]=\left(\mathrm{b}[\mathrm{i})^{1 / 3}\right. \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Copy | TMtxVec | $a[i]=b[i]$ | SSE3/SSE4/AVX |
| Ceil | TmtxVec | $\begin{aligned} & a[i]=\operatorname{Cos}(a[i]) \\ & a[i]=\operatorname{Cos}(b[i]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Cos | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\operatorname{Cos}(a[i]) \\ & a[i]=\operatorname{Cos}(b[i]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Cosh | TMtxVec | $a[i]=\operatorname{Cosh}(a[i])$ | SSE3/SSE4/AVX, |


|  |  | $\mathrm{a}[\mathrm{i}]=\operatorname{Cosh}(\mathrm{b}[\mathrm{i}])$ | SMP |
| :---: | :---: | :---: | :---: |
| Cot | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{Cot}(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Cot}(\mathrm{b}[\mathrm{i}]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Coth | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\operatorname{Coth}(a[i]) \\ & \mathrm{a}[i]=\operatorname{Coth}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Csc | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{Csc}(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Csc}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Csch | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\operatorname{Csch}(\mathrm{a}[\mathrm{i}) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Csch}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| CumSum | TDenseMtxVec | $\begin{aligned} & \mathrm{a}[i]=\mathrm{a}[\mathrm{i}]+\mathrm{A}[\mathrm{i}-1] \\ & \mathrm{a}[\mathrm{i}]=\mathrm{b}[\mathrm{i}]+\mathrm{A}[\mathrm{i}-1] \end{aligned}$ |  |
| Difference | TDenseMtxVec | $\begin{aligned} & \mathrm{a}[i]=\mathrm{A}[i+1]-\mathrm{a}[\mathrm{i}] \\ & \mathrm{a}[\mathrm{i}]=\mathrm{B}[\mathrm{i}+1]-\mathrm{b}[\mathrm{i}] \end{aligned}$ |  |
| Divide | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\mathrm{a}[\mathrm{i}] / \mathrm{b}[\mathrm{i}] \\ & \mathrm{a}[\mathrm{i}]=\mathrm{b}[\mathrm{i}] / \mathrm{c}[\mathrm{i}] \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { RCX } \\ & \hline \end{aligned}$ |
| DotProd | TDenseMtxVec | $\mathrm{S}=\operatorname{Sum}\left(\mathrm{a}[\mathrm{i}]^{*} \mathrm{~b}[\mathrm{i}]\right)$ | SSE3/SSE4/AVX |
| Exp | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{Exp}(\mathrm{a}[\mathrm{i}) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Exp}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Exp10 | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{Exp} 10(\mathrm{a}[\mathrm{i}) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Exp} 10(\mathrm{~b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Exp2 | TMtxVec | $\begin{aligned} & \text { a[i] }=\operatorname{Exp2(a[i])} \\ & a[i]=\operatorname{Exp2}(b[i)) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Frac | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\text { fractional part of } \mathrm{a}[\mathrm{i}] \\ & \mathrm{a}[\mathrm{i}]=\text { fractional part of } \mathrm{b}[\mathrm{i}] \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| IntPower | TMtxVec | $\begin{aligned} & a[i]=(a[i])^{i} \\ & a[i]=(b[i])^{i} \end{aligned}$ | SSE3/SSE4/AVX |
| Inv | TMtxVec | $\begin{aligned} & a[i]=1 / a[i] \\ & a[i]=1 / b[i] \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| InvCbrt | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\left(\mathrm{a}[\mathrm{i})^{-1 / 3}\right. \\ & \mathrm{a}[\mathrm{i}]=(\mathrm{b}[\mathrm{i}])^{-1 / 3} \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| InvSqrt | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=(\mathrm{a}[\mathrm{i}])^{-1 / 2} \\ & \mathrm{a}[\mathrm{i}]=\left(\mathrm{b}[\mathrm{i})^{-1 / 2}\right. \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| IsEqual | TMtxVec | $\mathrm{a}[\mathrm{i}]=?=\mathrm{b}[\mathrm{i}]$ |  |
| Ln | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\operatorname{Ln}(\mathrm{a}[i]) \\ & \mathrm{a}[i]=\operatorname{Ln}(\mathrm{b}[i]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Log10 | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\log 10(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\log 10(\mathrm{~b}[\mathrm{i}]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Log2 | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\log 2(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\log 2(\mathrm{~b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| LogN | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\log \mathrm{N}(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\log \mathrm{b}(\mathrm{~b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Max | TMtxVec | $\mathrm{S}=\operatorname{Max}(\mathrm{a}[\mathrm{i}])$ | SSE3/SSE4/AVX |
| MaxMin | TMtxVec | S1 = $\operatorname{Max}(a[i]), \mathrm{S} 2=\operatorname{Min}(a[i])$ | SSE3/SSE4/AVX |
| Mean |  | S = 1/Len*Sum(a[i]) | SSE3/SSE4/AVX |
| Min |  | $\mathrm{S}=\operatorname{Max}(\mathrm{a}[\mathrm{i}])$ | SSE3/SSE4/AVX |
| Mul | TMtxVec |  | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { RCX } \end{aligned}$ |
| Mul | TDenseMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\mathrm{a}[\mathrm{i}]{ }^{*} \mathrm{~b}[\mathrm{i}] \\ & \mathrm{a}[\mathrm{i}]=\mathrm{b}[\mathrm{i}]{ }^{*} \mathrm{c}[\mathrm{i}] \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { RCX } \end{aligned}$ |
| Normalize | TMtxVec | $\mathrm{a}[\mathrm{i}]=(\mathrm{a}[\mathrm{i}]-\mathrm{B}) / \mathrm{C}$ | SSE3/SSE4/AVX |
| Product | TMtxVec | S = Product(a[i]) | SSE3/SSE4/AVX |
| Power | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=(\mathrm{a}[i])^{\mathrm{B}} \\ & \mathrm{a}[\mathrm{i}]=(\mathrm{b}[\mathrm{i}])^{\mathrm{c}} \\ & \mathrm{a}[\mathrm{i}]=(\mathrm{B})^{\mathrm{c}[\mathrm{l}]} \\ & \mathrm{a}[\mathrm{i}]=(\mathrm{b}[\mathrm{i}])^{\mathrm{c}} \mathrm{li]} \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { RCX, } \\ & \text { SMP } \end{aligned}$ |
| Round | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\text { nearest integer to } \mathrm{a}[i] \\ & \mathrm{a}[\mathrm{i}]=\text { nearest integer to } \mathrm{b}[\mathrm{i}] \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Sec | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{Sec}(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Sec}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |


| Sech | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\operatorname{Sech}(\mathrm{a}[i]) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Sech}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| SetVal | TMtxVec | $\mathrm{a}[\mathrm{i}]=\mathrm{B}$ | SSE3/SSE4/AVX |
| SetZero | TMtxVec | $\mathrm{a}[\mathrm{i}]=0$ | SSE3/SSE4/AVX |
| Sgn | TMtxVec | $a[i]=\operatorname{signum}(a[i])$ |  |
| Sign | TMtxVec | $a[i]=-a[i]$ | SSE3/SSE4/AVX |
| Sin | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{Sin}(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Sin}(\mathrm{b}[\mathrm{i}]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMD } \end{aligned}$ |
| SinCos | TMtxVec | $b[i]=\operatorname{Sin}(a[i]), c[i]=\operatorname{Cos}(a[i])$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Sinh | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\operatorname{Sech}(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Sech}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| SinhCosh | TMtxVec | $\begin{aligned} & \mathrm{b}[i]=\operatorname{Sech}(\mathrm{a}[\mathrm{i}]), \quad \mathrm{c}[\mathrm{i}]= \\ & \operatorname{Sech}(\mathrm{a}[\mathrm{i}]) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Sqr | TMtxVec | $\begin{aligned} \mathrm{a}[\mathrm{i}] & =(\mathrm{a}[\mathrm{i}])^{2} \\ \mathrm{a}[\mathrm{i}] & =(\mathrm{b}[\mathrm{i}])^{2} \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Sqrt | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=(\mathrm{a}[\mathrm{i}])^{1 / 2} \\ & \mathrm{a}[\mathrm{i}]=\left(\mathrm{b}[\mathrm{i})^{1 / 2}\right. \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Sub | TMtxVec | $a[i]=a[i]-B$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { RCX } \end{aligned}$ |
| Sub | TDenseMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\mathrm{a}[\mathrm{i}]-\mathrm{b}[\mathrm{i}] \\ & \mathrm{a}[\mathrm{i}]=\mathrm{b}[\mathrm{i}]-\mathrm{c}[\mathrm{i}] \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { RCX } \end{aligned}$ |
| SubFrom | TDenseMtxVec | $a[i]=B-a[i]$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { RCX } \end{aligned}$ |
| Sum | TMtxVec | S = Sum(a[i]) | SSE3/SSE4/AVX |
| Tan | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{Tan}(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Tan}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Tanh | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{Tanh}(\mathrm{a}[\mathrm{i}) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{Tanh}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Trunc | TMtxVec | $a[i]=$ integer part of a[i] rounded to zero <br> $a[i]=$ integer part of b[i] rouned to zero | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| ThreshBottom ThreshTop | TMtxVec | Limit the upper or lower value range | SSE3/SSE4/AVX |

### 17.2 Statistical

| Kurtosis | TDenseMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\text { Kurtosis(a[i]) } \\ & \mathrm{a}[\mathrm{i}]=\text { Kurtosis }(\mathrm{b}[\mathrm{i}) \end{aligned}$ | SSE3/SSE4/AVX |
| :---: | :---: | :---: | :---: |
| Skewness | TDenseMtxVec | $\begin{aligned} & \mathrm{a}[i]=\text { Skewness }(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\text { Skewness }(\mathrm{b}[\mathrm{i}]) \end{aligned}$ | SSE3/SSE4/AVX |
| StdDev | TDenseMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\operatorname{StdDev}(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{StdDev}(\mathrm{b}[\mathrm{i}]) \end{aligned}$ | SSE3/SSE4/AVX |
| RMS | TDenseMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\mathrm{RMS}(\mathrm{a}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\operatorname{RMS}(\mathrm{b}[\mathrm{i}) \end{aligned}$ | SSE3/SSE4/AVX |
| Mean | TDenseMtxVec | $\mathrm{S}=\mathrm{Mean}(\mathrm{a}[\mathrm{i}])$ | SSE3/SSE4/AVX |

### 17.3 Complex number specific

| Expj | TMtxVec | $a[i]=\exp \left(-j^{*} w^{*} b[i]\right)$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Flip | TMtxVec | $\mathrm{a}[\mathrm{i}] \cdot \mathrm{Re}=\mathrm{b}[\mathrm{i}] \cdot \mathrm{Im}, \mathrm{a}[\mathrm{i}] \cdot \mathrm{Im}=\mathrm{b}[\mathrm{i}] \cdot \mathrm{Re}$ | SSE3/SSE4/AVX |
| CartToPolar/PolarToCart | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\text { CartToPolar(b[i], c[i]) } \\ & \mathrm{a}[\mathrm{i}]=\text { PolarToCart(b[i], c[i]) } \end{aligned}$ | SSE3/SSE4/AVX |
| CplxToReal/RealToCplx | TMtxVec | $\mathrm{a}[\mathrm{i}]=$ CplxToReal(b[i], c[i]) | SSE3/SSE4/AVX |


|  |  | $\mathrm{a}[\mathrm{i}]=$ RealToCplx(b[i], c[i]) |  |
| :---: | :---: | :---: | :---: |
| ExtendToComplex | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}] \cdot \mathrm{Re}=\mathrm{b}[\mathrm{i}] ; \mathrm{a}[\mathrm{i}] \cdot \operatorname{Re}=\mathrm{a}[\mathrm{i}] \\ & \mathrm{a}[\mathrm{i}] \cdot \mathrm{Im}=0 \end{aligned}$ | SSE3/SSE4/AVX |
| ImagPart | TMtxVec | $\mathrm{a}[\mathrm{i}]=\mathrm{b}[\mathrm{i}] . \mathrm{Im}$ | SSE3/SSE4/AVX |
| RealPart | TMtxVec | $a[i]=b[i] . \operatorname{Re}$ | SSE3/SSE4/AVX |
| Mull | TMtxVec | $\begin{aligned} & \mathrm{a}[i]=\mathrm{a}[i]^{* i} \\ & \mathrm{a}[\mathrm{i}]=\mathrm{b}[\mathrm{i}]^{*} \mathrm{i} \end{aligned}$ | SSE3/SSE4/AVX |
| ConjMul | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\mathrm{a}[\mathrm{i}]^{*} \operatorname{Conj}(\mathrm{~b}[\mathrm{i}]) \\ & \mathrm{a}[\mathrm{i}]=\mathrm{b}[\mathrm{i}]^{*} \operatorname{Conj}(\mathrm{c}[\mathrm{i}) \end{aligned}$ | SSE3/SSE4/AVX |
| Conj | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}]=\mathrm{a}[\mathrm{i}] \cdot \operatorname{Re}-\mathrm{a}[\mathrm{i}] \cdot \mathrm{Im} \\ & \mathrm{a}[\mathrm{i}]=\mathrm{b}[\mathrm{i}] \cdot \operatorname{Re}-\mathrm{b}[\mathrm{i}] \cdot \mathrm{Im} \end{aligned}$ | SSE3/SSE4/AVX |
| PhaseSpectrum | TMtxVec | $\mathrm{a}[\mathrm{i}]=\operatorname{Arctan} 2(\mathrm{~b}[\mathrm{i}] . \mathrm{Im} / \mathrm{b}[\mathrm{i}] . \mathrm{Re})$ | SSE3/SSE4/AVX |
| PowerSpectrum | TMtxVec | $a[i]=(\operatorname{sqr}(\mathrm{b}[\mathrm{i}] \cdot \mathrm{Re})+\operatorname{sqr}(\mathrm{b}[\mathrm{i}] . \operatorname{Im})$ ) | SSE3/SSE4/AVX |
| FlipConj | TMtxVec | $\begin{aligned} & \mathrm{a}[\mathrm{i}] \cdot \mathrm{Re}=\mathrm{b}[\mathrm{i}] \cdot \operatorname{Im} \\ & \mathrm{a}[\mathrm{i}] \cdot \mathrm{Im}=-\mathrm{b}[\mathrm{i}] \cdot \operatorname{Re} \end{aligned}$ | SSE3/SSE4/AVX |

### 17.4 Size, streaming and storage

CopyBinaryFromArray, CopyFromArray, LoadFromFile, CopyToArray, LoadFromStream, ReadHeader, ReadValues, SaveToFile, SaveToStream, SetCplx, SetDouble, SetInteger, Setlt, SetSingle, SizeToArray, WriteHeader, WriteValues, Size, Resize

### 17.5 FFT's

| Function | Class | Math expression | Features |
| :---: | :---: | :---: | :---: |
| FFT/IFFT | TDenseMtxVec TVec | $\begin{aligned} & \mathrm{A}=\mathrm{FFT}(\mathrm{~A}), \mathrm{A}=\operatorname{IFFT}(\mathrm{A}) \\ & \mathrm{A}=\mathrm{FFT}(\mathrm{~B}), \mathrm{A}=\operatorname{IFFT}(\mathrm{B}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| FFTFromReal | TDenseMtxVec TVec | $\begin{aligned} & \mathrm{A}=\mathrm{FFT}(\mathrm{~A}) \\ & \mathrm{A}=\mathrm{FFT}(\mathrm{~B}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| IFFTToReal | TDenseMtxVec TVec | $\begin{aligned} & \mathrm{A}=\operatorname{IFFT}(\mathrm{A}) \\ & \mathrm{A}=\operatorname{IFFT}(\mathrm{B}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| FFT1D/IFFT1D | TMtx | $\begin{aligned} & \mathrm{A}(\mathrm{i})=\mathrm{FFT}(\mathrm{~A}(\mathrm{i})), \mathrm{A}(\mathrm{i})=\operatorname{IFFT}(\mathrm{A}(\mathrm{i})) \\ & \mathrm{A}(\mathrm{i})=\mathrm{FFT}(\mathrm{~B}(\mathrm{i})), \mathrm{A}(\mathrm{i})=\operatorname{IFFT}(\mathrm{B}(\mathrm{i})) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| FFT2D | TMtx | $\begin{aligned} & \mathrm{A}=\mathrm{FFT} 2 \mathrm{D}(\mathrm{~A}) \\ & \mathrm{A}=\mathrm{FFT} 2 \mathrm{D}(\mathrm{~B}) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| FFT2DFromReal | TMtx | $\begin{aligned} & \mathrm{A}=\mathrm{FFT} 2 \mathrm{D}(\mathrm{~A})) \\ & \mathrm{A}=\mathrm{FFT} 2 \mathrm{D}(\mathrm{~B})) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| FFT1DFromReal | TMtx | $\begin{aligned} & \mathrm{A}(\mathrm{i})=\mathrm{FFT}(\mathrm{~A}(\mathrm{i})) \\ & \mathrm{A}(\mathrm{i})=\mathrm{FFT}(\mathrm{~B}(\mathrm{i})) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| IFFT1DToReal | TMtx | $\begin{aligned} \mathrm{A}(\mathrm{i}) & =\operatorname{IFFT}(\mathrm{A}(\mathrm{i})) \\ \mathrm{A}(\mathrm{i}) & =\operatorname{IFFT}(\mathrm{B}(\mathrm{i})) \end{aligned}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| DCT | TVec | $\mathrm{A}=\mathrm{DCT}(\mathrm{B})$ | SSE3/SSE4/AVX |
| IDCT | TVec | $\mathrm{A}=\operatorname{IDCT}(\mathrm{B})$ | SSE3/SSE4/AVX |

### 17.6 Linear algebra

| Function | Class | Math expression | Features |
| :--- | :--- | :--- | :--- |
| LUSolve | TMtx | $A^{*} X=\mathrm{B}$, solves for X | SSE3/SSE4/AVX, <br> SMP |
| LQRSolve | TMtx | Least squares soltuion | SSE3/SSE4/AVX, <br> SMP |
| SVDSolve | TMtx | Singular value solution | SSE3/SSE4/AVX, <br> SMP |
| Eig | TMtx | Eigvalues and eigen vectors | SSE3/SSE4/AVX, <br> SMP |
| EigGen | TMtx | Generalized eigen values | SSE3/SSE4/AVX, |


|  |  |  | SMP |
| :---: | :---: | :---: | :---: |
| Transp | TMtx | Transpose | SSE3/SSE4/AVX |
| Adjung | TMtx | Adjungate | SSE3/SSE4/AVX |
| Cholesky | TMtx | Cholesky factorization | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Determinant | TMtx | Determinant | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Inverse | TMtx | $\mathrm{A}^{-1}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| LU | TMtx | LU factorization | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| LQR | TMtx | LQ and QR factorization | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| SVD | TMtx | SVD decomposition | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| Mul | TMtx | Matrix multiply | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| MtxFunction | TMtx | Matrix function: $\mathrm{A}=\mathrm{Fun}(\mathrm{B})$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| MtxSqrt | TMtx | $\mathrm{A}^{-0.5}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| MtxPower | TMtx | $\mathrm{A}^{\mathrm{p}}$ | $\begin{aligned} & \text { SSE3/SSE4/AVX, } \\ & \text { SMP } \end{aligned}$ |
| MtxIntPower | TMtx | $\mathrm{A}^{\mathrm{i}}$ | SSE3/SSE4/AVX |
| TensorProd | TMtx | $\begin{aligned} & \text { aMtx }=\text { alfa*bMtx*Vec } \\ & \text { aMtx }=\text { alfa*Vec*Mtx } \\ & \text { aMtx }=\text { Vec } 1 \times \text { Vec2 } \end{aligned}$ | SSE3/SSE4/AVX |
| AddTensorProd | TMtx | Mtx = alfa*Vec1 x Vec2 + Mtx. | SSE3/SSE4/AVX |
| Sylvester | TMtx | The Sylvester equation | SSE3/SSE4/AVX |

### 17.7 Matrix conversions

| Function | Class | Math expression | Features |
| :--- | :--- | :--- | :--- |
| BandedToDense | TMtx | Banded matrix to dense |  |
| DenseToBanded | TMtx | Dense matrix to banded |  |

### 17.8 Miscellaneous matrix routines

| Function | Class | Math expression | Features |
| :--- | :--- | :--- | :--- |
| Diag | TMtx | Matrix diagonal |  |
| Eye | TMtx | Matrix with 1's on main diagonal. |  |
| Concat, <br> ConcatHorz, <br> ConcatVert | TMtx | Concatenate matrices | SSE3/SSE4/AVX |
| FlipVer, FlipHor | TMtx | Flip matrices |  |
| Kron | TMtx | Kronecker product | SSE3/SSE4/AVX |
| LowerTriangle, <br> UpperTriangle | TMtx | SSE3/SSE4/AVX |  |
| Norm1, <br> NormFro, <br> NormInf | TMtx | Special matrices | SSE3/SSE4/AVX |
| Pascl, <br> VanderMond, <br> Toeplitz | TMtx | TMtx | Rotate the matrix |
| Rotate90 | TMtx | Sumies row/column | SSE3/SSE4/AVX |
| SetCol, SetRow | TMtx | SSE3/SSE4/AVX |  |
| SumCols, |  |  |  |


| SumRows |  |  |  |
| :--- | :--- | :--- | :--- |
| MeanCols, <br> MeanRows | TMtx | Average of columns or rows | SSE3/SSE4/AVX |

